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Stagnation Point Flow over a Stretching Sheet with Newtonian Heating

(Aliran Titik Genangan pada Helaian Meregang dengan Pemanasan Newtonan)

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ABSTRACT

In this study, the numerical solution of stagnation point flow over a stretching surface, generated by Newtonian heating in which the heat transfer from the surface is proportional to the local surface temperature is considered. The transformed boundary layer equations are solved numerically using the shooting method. Numerical solutions are obtained for the local heat transfer coefficient, the surface temperature and the temperature profiles. The features of the flow and heat transfer characteristics for various values of the Prandtl number, stretching parameter and conjugate parameter are analyzed and discussed.

Keywords: Newtonian heating; numerical solution; stagnation point flow; stretching sheet

ABSTRAK

Dalam kajian ini, penyelesaian berangka bagi masalah aliran titik genangan pada permukaan meregang yang dijanakan oleh pemanasan Newtonan, iaitu pemindahan haba daripada permukaan berkadar langsung dengan suhu permukaan setempat dipertimbangkan. Persamaan lapisan sempadan terjelma diselesaikan secara berangka dengan kaedah tembakan. Penyelesaian berangka diperoleh bagi pekali pemindahan haba setempat, suhu permukaan dan profil suhu. Ciri-ciri aliran dan pemindahan haba bagi pelbagai nilai nombor Prandtl, parameter regangan dan parameter konjugat dianalisis dan dibincangkan.

Kata kunci: Aliran titik genangan; helaian meregang; pemanasan Newtonan; penyelesaian berangka

INTRODUCTION

Problems related to convection boundary layer flows are important in engineering and industrial activities. Such flows are applied to manage thermal effects in many industrial outputs, for example in electronic devices, computer power supply and also in engine cooling system such as heatsink in car radiator. Sakiadis (1961) was the first to study the boundary layer flow on a continuous solid surface moving at constant speed. Due to entrainment of the ambient fluid, this boundary layer flow is quite different from Blasius flow past a flat plate. Sakiadis's theoretical predictions for Newtonian fluids were later corroborated experimentally by Tsou et al. (1967). Flow of a viscous fluid past a stretching sheet is a classical problem in fluid dynamics. Crane (1970) was the first to study convection boundary layer flow over a stretching sheet. The heat and mass transfer on a stretching sheet with suction or blowing was investigated by Gupta and Gupta (1977). They considered an isothermal moving plate and obtained the temperature and concentration distributions. Chen and Char (1988) studied laminar boundary layer flow and heat transfer from a linearly stretching, continuous sheet subjected to suction or blowing. Two cases were considered; moving plate with prescribed wall temperature and heat flux. Ishak et al. (2007, 2008, 2009) studied the MHD stagnation point flow towards a stretching sheet, mixed convection towards vertical and continuosly stretching sheet and post stagnation-point towards vertical and linearly stretching sheet. This type of problem was then extended to viscous fluids, viscoelastic fluids or micropolar fluids by many investigators by considering the usually applied boundary conditions, either prescribed wall temperature or prescribed wall heat flux.

On the other hand, Merkin (1994) has shown that, in general, there are four common heating processes specifying the wall-to-ambient temperature distributions, namely, (1) constant or prescribed wall temperature; (2) constant or prescribed surface heat flux; (3) conjugate conditions, where heat is supplied through a bounding surface of finite thickness and finite heat capacity. The interface temperature is not known a priori but depends on the intrinsic properties of the system, namely the thermal conductivity of the fluid or solid; and (4) Newtonian heating, where the heat transfer rate from the bounding surface with a finite heat capacity is proportional to the local surface temperature and is usually termed conjugate convective flow.

Generally, in modeling the convection boundary layer flow, the boundary conditions that were usually applied are (1) and (2). However, the Newtonian heating condition, (4) has been used only quite recently by Lesnic et al. (2004); Merkin (1994) and Pop et al. (2000) to study the free convection boundary layer flow over vertical and horizontal surfaces embedded in a porous medium. The asymptotic solution near the leading edge and the full numerical solution along the whole plate domain have been obtained numerically, whilst the asymptotic solution for downstream along the plate has been obtained analytically. Recently Salleh et al. (2009, 2010, 2011b) and Salleh and Nazar (2010) studied the forced convection boundary layer flow at a forward stagnation point with Newtonian heating as well as the forced and free convection boundary layer flow over a horizontal circular cylinder with Newtonian heating.

The situation with Newtonian heating arises in what are usually termed conjugate convective flows, where the heat is supplied to the convective fluid through a bounding surface with a finite heat capacity. This configuration occurs in many important engineering devices, for example in heat exchangers where the conduction in solid tube wall is greatly influenced by the convection in the fluid flowing over it. This configuration also occurs for conjugate heat transfer around fins where the conduction within the fin and the convection in the fluid surrounding it must be simultaneously analyzed in order to obtain the vital design information and also in convection flows set up when the bounding surfaces absorb heat by solar radiation.

The aim of this study was to investigate the problem of stagnation point flow over a stretching sheet with Newtonian heating (NH). The governing nonlinear partial differential equations are first transformed into a system of ordinary differential equations by a similarity transformation, before being solved numerically using the shooting method. To the best of our knowledge this problem has not been considered before, so that the reported results are new.

Mathematical Formulation

Consider a steady two-dimensional stagnation-point flow over a stretching/shrinking plate immersed in an incompressible viscous fluid of ambient temperature, T_∞. It is assumed that the external velocity $u_e(x)$ and the stretching velocity $u_w(x)$ are of the forms $u_e(x) = ax$ and $u_w(x) = bx$ where a and b are constants. It is further assumed that the plate is subjected to a Newtonian heating proposed by Merkin (1994). The boundary layer equations are:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0. \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = u_e\frac{du_e}{dx} + \nu\frac{\partial^2 u}{\partial y^2}, \tag{2}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2}, \tag{3}$$

subject to the boundary conditions:

$$u = u_w(x), \quad v = 0, \quad \frac{\partial T}{\partial y} = -h_s T \quad \text{at } y = 0$$

$$u = u_e(x), \quad T \to T_\infty \quad \text{as} \quad y \to \infty, \tag{4}$$

where u and v are the velocity components along the x and y directions, respectively. Further, T is the fluid temperature in the boundary layer, ν is the kinematic viscosity, α is the thermal diffusivity and h_s is the heat transfer coefficient.

We introduce now the following similarity variables (Salleh et al. 2010):

$$\eta = \left(\frac{a}{\nu}\right)^{1/2} y, \quad \psi = (a\nu)^{1/2}\, xf(n), \quad \theta(\eta) = \frac{T - T_\infty}{T_\infty}, \tag{5}$$

where ψ is the stream function defined as $u = \frac{\partial \psi}{\partial y}$ and $v = -\frac{\partial \psi}{\partial x}$, which identically satisfies Equation (1). Thus, we have:

$$u = axf'(\eta), \quad v = -(a\nu)^{1/2} f(\eta), \tag{6}$$

where prime denotes differentiation with respect to η. Substituting (5) and (6) into (2) and (3), we obtain the following nonlinear ordinary differential equations:

$$f''' + ff'' + 1 - f'^2 = 0. \tag{7}$$

$$\frac{1}{\Pr}\theta'' + f\theta' = 0, \tag{8}$$

where $\Pr = \frac{\nu}{\alpha}$ is the Prandtl number. The boundary conditions (4) become:

$$f(0) = 0, \quad f'(0) = \varepsilon, \quad \theta'(0) = -\gamma(1 + \theta(0)) \tag{9}$$

$$f'(\eta) \to 1, \quad \theta(\eta) \to 0 \quad \text{as} \quad \eta \to \infty, \tag{10}$$

where $\varepsilon = \frac{b}{a}$ is the stretching parameter. Further, $\gamma = h_s \left(\frac{\nu}{a}\right)^{1/2}$ is the conjugate parameter for Newtonian heating. It is noticed that $\gamma = 0$ is for the insulated plate and $\gamma \to \infty$ is when the surface temperature remains constant. The physical quantities of interest are the skin friction coefficient C_f and the local Nusselt number Nu_x which are given by:

$$C_f = \frac{\tau_w}{\rho u_e^2}, \quad Nu_x = \frac{xq_w}{k(T_w - T_\infty)}, \tag{11}$$

where ρ is the fluid density. The surface shear stress τ_w and the surface heat flux q_w are given by:

$$\tau_w = \mu\left(\frac{\partial u}{\partial y}\right)_{y=0}, \quad q_w = -k\left(\frac{\partial T}{\partial y}\right)_{y=0}, \tag{12}$$

with $\mu = \rho\nu$ and k being the dynamic viscosity and the thermal conductivity, respectively. Using the similarity variables in (5) give:

$$C_f \, \mathrm{Re}_x^{1/2} = f''(0), \qquad Nu_x \, \mathrm{Re}_x^{-1/2} = -\frac{\theta'(0)}{\theta(0)}, \tag{13}$$

where $\mathrm{Re}_x = \frac{u_e x}{v}$ is the local Reynolds number and Nu_x is the local Nusselt number.

Results and Discussion

Equations (7) and (8) subject to the boundary conditions (9) and (10) were solved numerically using the shooting method with three parameters considered, namely the Prandtl number Pr, the conjugate parameter γ and the stretching parameter ε. In order to validate the efficiency of the method used, the comparison of the values of the surface temperature $\theta(0)$ and heat transfer coefficient $-\theta'(0)$ has been made. Due to the decoupled boundary layer (7) and (8), for $\varepsilon = 0$, it is found that there is a unique value of the skin friction coefficient, $f''(0) = 1.23258766$, which is in very good comparison with the classical value $f''(0) = 1.232588$ by Hiemenz (1911). Table 1 presents the comparison between the present results with the previously reported results by Salleh (2011) for various values of the Prandtl number Pr when $\gamma = 1$ and $\varepsilon = 0$. Also, it is found that they are in good agreement.

TABLE 1. Comparison between the present solution of (7) and (8) with previously published results when $\gamma = 1$ and $\varepsilon = 0$

Pr	Salleh (2011)		Present	
	$\theta(0)$	$-\theta'(0)$	$\theta(0)$	$-\theta'(0)$
5	23.0042	24.0042	23.0239	24.0239
7	5.6872	6.6872	5.6062	6.6062
10	2.9226	3.9226	2.9516	3.9516
100	0.6866	1.6866	0.5034	1.5034
1000	0.2593	1.2593	0.1809	1.1809

Tables 2 and 3 present the values of $\theta(0)$ and for various values of ε when $\gamma = 1$ and Pr = 5, and various values of Pr when $\gamma = 1$ and $\varepsilon = 1$, respectively. It is noticed that as ε or Pr increases, the values of $\theta(0)$ and $-\theta'(0)$ decrease. Table 4 presents the values of $\theta(0)$ and $-\theta'(0)$ for various values of γ when Pr = 5 and $\varepsilon = 1$. It is found that the values of $\theta(0)$ and $-\theta(0)$ increase as γ increases.

Figure 1 illustrates the variation of the surface temperature $\theta(0)$ with ε when $\gamma = 1$ and Pr = 5. To get a physically acceptable solution, ε must be greater than or equal to a critical value, say ε_c, i.e. $\varepsilon \geq \varepsilon_c$. It can be seen from this figure that $\theta(0)$ becomes large (unbounded) as ε approaches the critical value $\varepsilon_c = -0.0480$. Figure 2 shows the variation of the surface temperature $\theta(0)$ with Prandtl number Pr, when $\gamma = 1$ and $\varepsilon = 1$. Also, to get a physically acceptable solution, Pr must be greater than or equal to a critical value, say Pr_c i.e. $\mathrm{Pr} \geq \mathrm{Pr}_c$. It can be seen from this figure that $\theta(0)$ becomes large (unbounded) as Pr approaches the critical value, $\mathrm{Pr}_c \cong 1.5740$. Figure 3 shows the variation of the surface temperature $\theta(0)$ with conjugate parameter γ when $\varepsilon = 1$ and Pr = 5. Different from the case illustrated in Figures 1 and 2, to get a physically acceptable solution, the conjugate parameter γ must be less than or equal to a critical value, say γ_c i.e. $\gamma \leq \gamma_c$. It can be seen from this figure that $\theta(0)$ becomes large (unbounded) as γ approaches the critical value, $\gamma_c \cong 1.7808$.

TABLE 2. Values of $\theta(0)$ and $-\theta'(0)$ from (7) and (8) for various values of ε when $\gamma = 1$ and Pr = 5

ε	$\theta(0)$	$-\theta'(0)$
0	23.0239	24.0239
2	0.7442	1.7442
4	0.4533	1.4533
5	0.3900	1.3900
10	0.2505	1.2505
100	0.0681	1.0681
1000	0.0206	1.0206

TABLE 3. Values of $\theta(0)$ and $-\theta'(0)$ from (7) and (8) for various values of Pr when $\gamma = 1$ and $\varepsilon = 1$

Pr	$\theta(0)$	$-\theta'(0)$
1.6	108.0728	109.0728
2	7.7894	8.7894
4	1.6785	2.6785
5	1.2753	2.2753
7	0.9001	1.9001
10	0.6565	1.6565
100	0.1433	1.1433
1000	0.0413	1.0413

TABLE 4. Values of $\theta(0)$ and $-\theta'(0)$ from (7) and (8) for various values of γ when Pr = 5 and $\varepsilon = 1$

γ	$\theta(0)$	$-\theta'(0)$
1	1.2753	2.2753
1.2	2.0544	3.6653
1.4	3.6447	6.5026
1.5	5.2794	9.4191
1.6	8.6898	15.5034

Figure 4 presents the temperature profiles for various values of Pr. It is found that as Pr increases, the temperature in the boundary layer decreases and the thermal boundary layer thickness also decreases. This is because for small values of the Prandtl number, the fluid is highly thermal conductive. Physically, if Pr increases, the thermal diffusivity decreases and these phenomena lead to the decreasing of energy ability that reduces the thermal

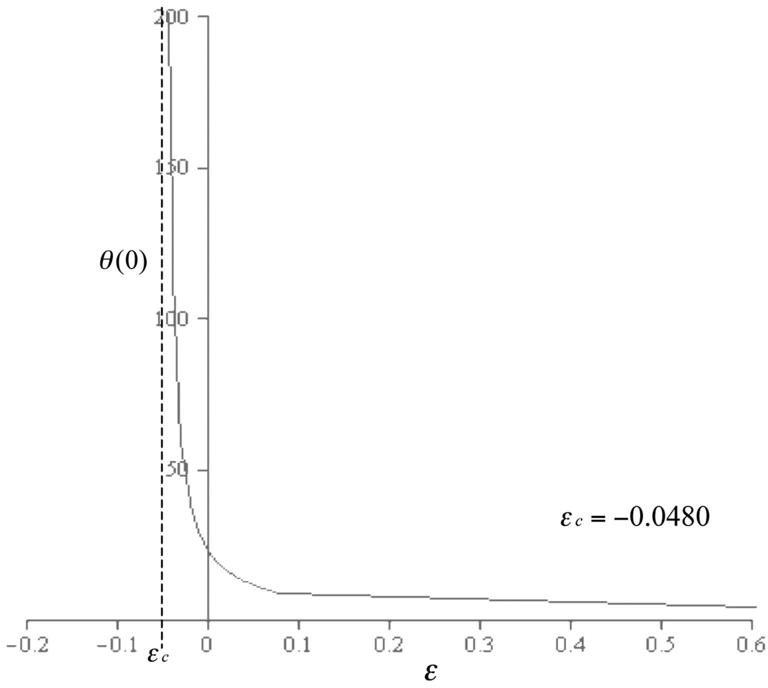

FIGURE 1. Variation of the surface temperature $\theta(0)$ with ε when $\gamma = 1$ and Pr $\cong$ 5

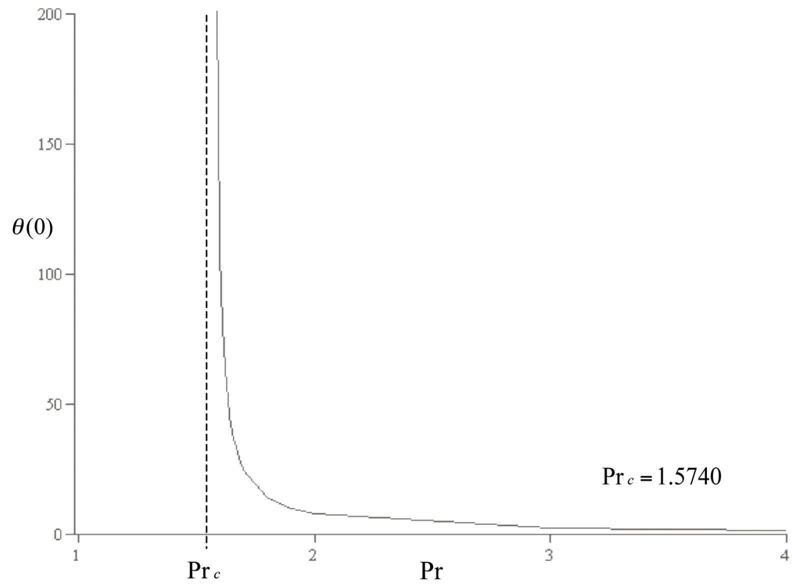

FIGURE 2. Variation of the surface temperature θ(0 with Prandtl number Pr when $\gamma = 1$ and $\varepsilon = 1$

boundary layer. The temperature profiles with various values of ε are presented in Figure 5 and it is found again that as ε increases, the temperature decreases, and the thermal boundary layer thickness also decreases, similar to Figure 4.

Lastly, the temperature profiles presented in Figure 6 show that when the value of the conjugate parameter γ decreases it is found that the temperature also decreases, contrary to the temperature profiles with various values of Pr and ε in Figures 4 and 5.

CONCLUSION

In this paper we have theoretically and numerically studied the problem of stagnation point flow over a stretching sheet with Newtonian heating condition. We

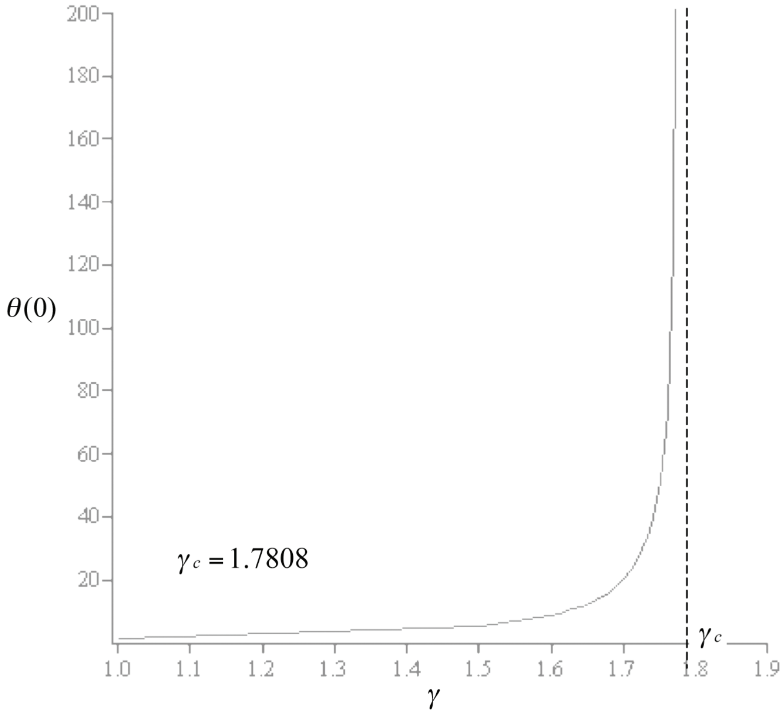

FIGURE 3. Variation of the surface temperature θ(0) with conjugate parameter γ when ε = 1 and Pr ≅ 5

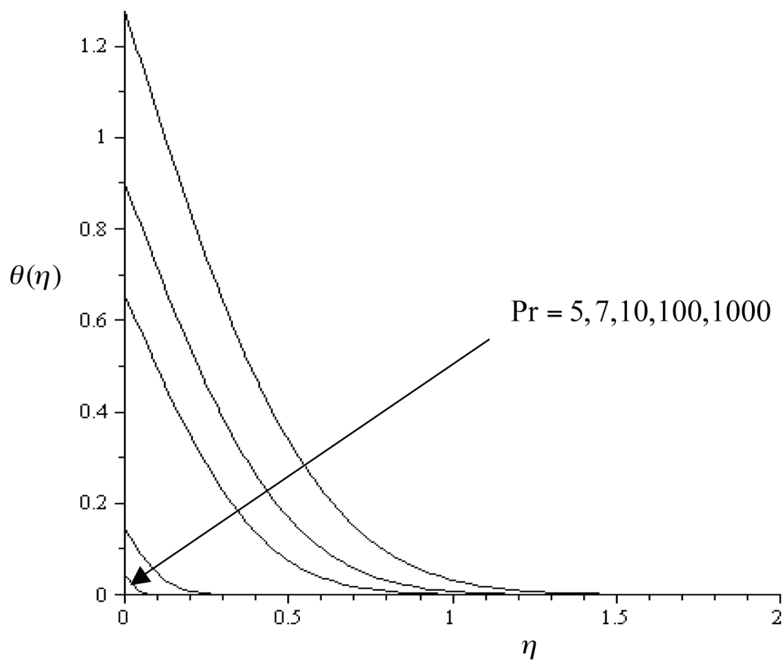

FIGURE 4. Temperature profiles $\theta(\eta)$ for various values of Pr when γ = 1 and ε = 1

can conclude that, to get a physically acceptable solution; Pr must be greater than or equal to Pr c (critical value of Pr) depending on γ and ε, ε must be greater than or equal to ε_c (critical value of ε) depending on γ and Pr and γ must be less than or equal to γ_c (critical value of γ) depending on ε and Pr.

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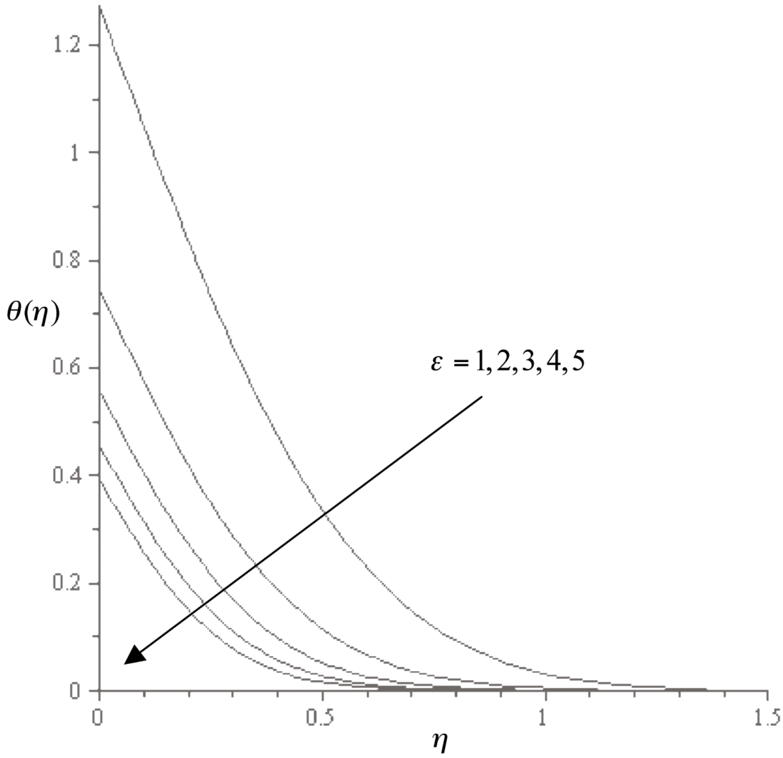

FIGURE 5. Temperature profiles $\theta(\eta)$ for various values of ε when $\gamma = 1$ and Pr = 5

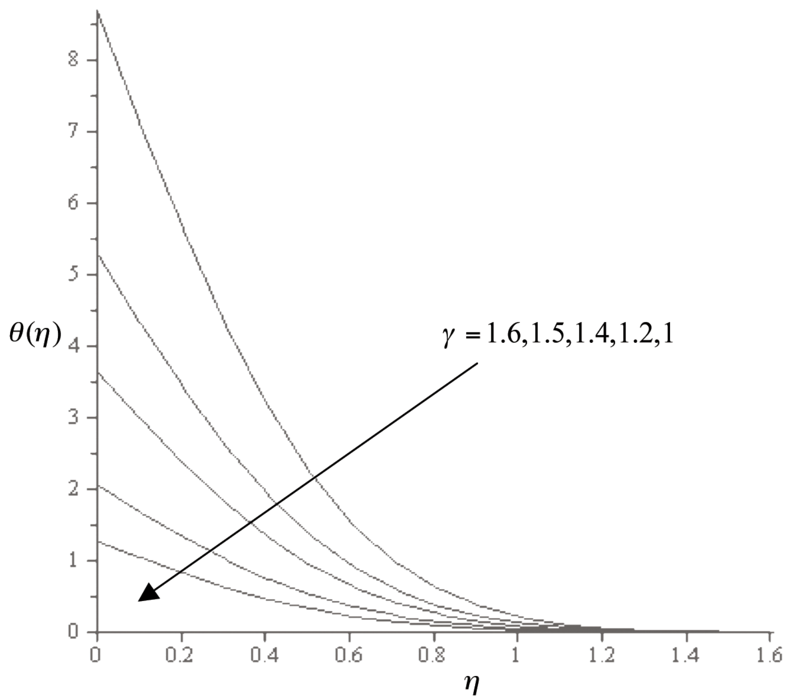

FIGURE 6. Temperature profiles $\theta(\eta)$ for various values of γ when $\varepsilon = 1$ and Pr = 5

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